ANSWER TO QUERY IN NO. 3.

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QUERY.—"The strain on the internal surface of a steam boiler tending to produce longitudinal rupture, is, per unit of length, equal to the product of the elastic pressure of the steam per unit of surface multiplied by the diameter (?) of the boiler. What should be the relative thickness of the flat ends of a boiler, so that, without bracing, they are just strong enough to withstand the maximum strain that the boiler will bear."

In the first place the word *diameter* used above should be *radius*.*

Let p = pressure per unit of surface.

h = thickness of end plate.

f = modulus of rupture for breaking across.

f_1 = " " " " longitudinal tension.

The formula for the strength of a rectangular beam uniformly loaded is $P = p\,b\,l = \frac{4}{3} . \frac{f\,b\,h^2}{l}$.

We may consider the portion of the end plate whose length is $2\,x$, breadth $d\,y$, and thickness h, as such a beam. The pressure on this elementary beam will be $= 2\,p\,x\,d\,y$.

$$\therefore \quad 2\,p \int_0^r x\,d\,y = \frac{4}{3} f\,h^2 \int_0^r \frac{d\,y}{2\,x}, \quad \text{or}$$

*Because the strength of the metal is supposed to be uniform, we may assume that rupture occurs simultaneously along two opposite sides of the boiler, (for when the force is sufficient to produce rupture at any one point, it is obviously just sufficient to produce rupture at the opposite point).

Imagine a plane, therefore, to separate the cavity of the boiler longitudinally into two equal parts; we may assume that rupture takes place along the lines that limit the two edges of this imaginary plane; and the force which produces these two lines of rupture is obviously measured by the total reaction of the steam in either half of the boiler, on the above named imaginary plane. Hence, though it is true that to produce a single line of rupture would require a force equivalent to the elastic pressure of the steam per unit of surface multiplied by the *radius* of the boiler, yet it is equally true that the force which *produces* rupture is represented by the total reaction of the steam in either half of the boiler on the above named imaginary plane, and is therefore "equal to the product of the elastic pressure of the steam, per unit of surface, multiplied by the *diameter* of the boiler." Practically, however, it is immaterial whether we conceive the boiler to be ruptured at two opposite points as a result of the total pressure, or whether we regard only a single line of rupture as resulting from half the pressure.—ED.

$$3p\int_o^r x\,dy = fh^2\int_o^r \frac{dy}{x}.$$

But from the equation of the circle we have

$$x = \sqrt{r^2 - y^2}.$$

$$\therefore \qquad 3p\int_o^r dy\sqrt{r^2-y^2} = fh^2\int_o^r \frac{dy}{\sqrt{r^2-y^2}}.$$

Whence by integration,

$$3p\,.\frac{\pi r^2}{4} = fh^2.\frac{\pi}{2}.$$

$$\therefore \qquad h = r\sqrt{\frac{3p}{2f}}.$$

The formula for the thickness of the cylindrical portion of the boiler is, if $t =$ the thickness,

$$t = \frac{pr}{f_1}.$$

Comparing these two expressions we see that

$$\frac{h}{t} = r\sqrt{\frac{3p}{2f}} \div \frac{pr}{f_1} = \frac{rf_1}{rp}\sqrt{\frac{3p}{2f}} = f_1\sqrt{\frac{3}{2pf}}.$$

This formula gives the thickness h much greater than t, as must evidently be the case, though since p is in the denominator this difference will diminish as p increases.

[We have inserted the foregoing answer to "Query" in No. 3 because it is the only answer that has been received. We think however that the investigation is defective, because it is virtually assumed that the sum of the capacities of the elementary beams to support a load uniformly distributed on each, divided by the area of the end of the boiler, is its capacity per unit of surface. This assumption has not been proved and is probably not true. For, though the lateral connection of the elementary beams serves to equalize their capacity to some extent, yet it is not likely that it exactly proportions the capacity of each elementary beam to its length, as it must if the result obtained be rigorously true. On the contrary, it is highly improbable that if equally pressed over the whole of its internal surface the end of a boiler would be equally liable to yield at every point of its surface. The subject is one of practical importance, however, and as we are not aware that it has been fully discussed heretofore, we would be pleased to have it thoroughly investigated in our pages.]